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Abstract

It is a well established idea that Bertrand competition is more efficient in welfare terms than Cournot competition regardless of the degree of substitutability or complementarity of the commodities produced by the firms. In this paper I show that, introducing incomplete information about rivals' costs of production this conclusion does not always hold: in a homogeneous duopoly, the Bertrand price (aggregate output) is higher (lower) than the Cournot one if both firms have low costs of production and the costs are uniformly distributed.

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1 Introduction

The Cournot (1838) and Bertrand (1883) models are cornerstones in the modern theory of oligopoly. In the former firms' strategic variable is the quantity of output to produce while in the latter firms choose the price. Interestingly, Bertrand competition has always been considered as more efficient in welfare terms than Cournot competition because it leads to lower prices and larger quantities (see for example Shubik (1980), Singh and Vives (1984) and Vives (1985)). Indeed, if we assume that firms produce a homogeneous product at a common constant marginal cost, Bertrand competition will lead to a price equal to the marginal cost while Cournot competition will lead to a price which is intermediate between the competitive and the monopolistic price. If, to the contrary, we assume that firms produce differentiated products, then, Bertrand price will be above the marginal cost but it will be again lower than the corresponding Cournot price. Therefore, consumer surplus and total surplus are always higher in Bertrand competition than in Cournot competition. Furthermore, profits in Cournot competition are higher, equal or smaller than in Bertrand competition if the goods are substitutes, independent or complements.[1]

However, Singh and Vives (1984) state that the conclusion that Bertrand competition is more efficient than Cournot competition is not correct "if one considers supergame equilibria. Price-setting supergame equilibria may support higher prices than quantity-setting equilibria for either homogeneous or differentiated products. See Brock and Scheinkman (1981) and Deneckere (1983)". That is, Singh and Vives restrict the validity of the conclusion to the class of static games only. Moreover, Vives (1984), analysing an incomplete information setting where firms receive signals about the uncertain demand, proves that the Bertrand Bayesian-Nash price (quantity) is, again, lower (higher) than the Cournot Bayesian-Nash one.

In this paper I argue that introducing incomplete information about

[1] Related results can be found in Cheng (1984, 1985) and Hathaway and Rickard (1979).

rivals' costs of production leads to completely different results. Indeed, I show that in a homogeneous duopoly in which each firm knows the value of its own marginal cost and the distribution function of its rival's one, in equilibrium, the Bertrand price (quantity) might be higher (lower) than the Cournot price (quantity). This will be the case -rather surprisingly- when both firms are relatively efficient, that is, have low costs. The intuition for this result is that when both firms have low costs, they will both produce a relatively large quantity in the Cournot game so that the price will be relatively low. To the contrary, in the Bertrand game, only one firm will produce in equilibrium and will sell at a high price-cost margin as long as its cost is low. Moreover, *ex ante* expected profits, i.e. before the game is actually played and the true costs revealed, are always higher in the Bertrand game regardless the value of each firm's cost. This conclusion is exactly the opposite of the one obtained by Vives (1984) in his model with uncertain demand. Indeed, Vives shows that firms' expected profits are always higher in the Cournot game. Finally, while the *ex post* profit of the less efficient firm is generally positive in the Cournot game and always zero in the Bertrand game, the *ex post* profit of the most efficient firm is very likely to be higher in the latter.

The paper is organised as follows. In Section 2, I analyse the Bertrand and Cournot static games with incomplete information in an industry with n firms calculating the equilibrium prices and quantities and also firms' *ex ante* and *ex post* profits. In Section 3, I make comparisons between these equilibrium values in the special case of a duopoly because a higher number of firms would complicate the analysis without affecting the results qualitatively.

2 Cournot and Bertrand models with incomplete information.

In both models I will use the following assumptions.

A1) In the industry there are n firms producing a homogeneous product;

A2) The demand function is a linear function of the price: i.e. $Q = 1 - p$ where $Q = \sum_{i=1}^{n} q_i$ is the aggregate quantity and p is the price;

A3) The cost function for firm i is $C_i(q_i) = c_i q_i$; i.e. there are no fixed costs and the marginal cost is constant;

A4) The marginal cost c_i is independently and uniformly distributed on $[0, 1]$;

A5) Each firm knows the value of its own cost, but only knows the distribution function of its rivals' unit costs;

A6) Firms meet only once in the market and they choose the value of their strategic variable (i.e. the price in the Bertrand game and the quantity in the Cournot game) simultaneously and non-cooperatively.

Given the above assumptions both the Bertrand and the Cournot games are static games of incomplete information. Therefore, the relevant concept of equilibrium will be the Bayesian-Nash equilibrium. In the next two subsections I compute the equilibria for each of the two games under analysis.

2.1 The Bertrand game.

The Bertrand Bayesian-Nash equilibrium when rivals' costs are unknown has been recently characterised by Spulber (1995). Using more general assumptions than those described above (i.e. the demand function is not necessarily linear and the costs are not necessarily drawn from a uniform distribution) he proves that the static Bertrand game of incomplete information has a unique symmetric equilibrium pricing strategy $p^*(c)$ which is increasing, differentiable and solves a system given by a differential equation and two boundary conditions. However, because of the generality of the assumptions made he does not actually solve for the equilibrium. Using the more specific assumptions A1-A6 and much simpler technicalities I now show that it is possible to derive an exact solution.

Recall that in a static Bayesian game, a strategy for player i is a function from types to actions. Hence, in our game, a strategy for firm i is a function $p_i(c_i)$ which specifies a price p_i for each possible value of the marginal cost c_i. Suppose, then, that all i's rivals adopt the same strategy $p(c_j)$ with $j = 1, ..., n$ and $i \neq j$.

Firm i's market demand is

$$q_i^B = \begin{cases} 1 - p_i & \text{if } p_i < \widehat{p}_{-i} \\ \frac{1-p_i}{m} & \text{if } p_i = \widehat{p}_{-i} \\ 0 & \text{if } p_i > \widehat{p}_{-i} \end{cases}$$

where

$\widehat{p}_{-i}$ is the smallest price in the set of the equilibrium prices chosen by all i's rivals using function $p(.)$;

$m \in [2, n]$ is the number of firms that charge $\widehat{p}_{-i}$.

In order to choose its optimal price, firm i will solve the following maximisation problem given the value of its cost of production c_i:

$$\max_{p_i} (1\text{-}p_i)(p_i\text{-}c_i) \Pr(p_i{<}\widehat{p}_{-i}) + \frac{(1\text{-}p_i)(p_i\text{-}c_i)}{m} \Pr(p_i{=}\widehat{p}_{-i}) + 0 \Pr(p_i{>}\widehat{p}_{-i}). \tag{1}$$

Now, since the marginal costs are distributed along a continuous interval, we have

$$\Pr(c_i = c_j) = 0 \qquad \forall i, j.$$

Supposing that the strategy $p(.)$ adopted by all i's rivals is a strictly monotone and differentiable function of the marginal cost, then also the prices will be uniformly and independently distributed. So

$$\Pr(p_i = p_j) = 0 \qquad \forall i, j.$$

Hence, we can rewrite (1) in the following way:

$$\max_{p_i} (1 - p_i)(p_i - c_i) \Pr (p_i < \widehat{p}_{-i}). \tag{2}$$

Now, since all firms different from i adopt the same strategy $p(.)$ and since the costs are drawn independently, we have:

$$\Pr (p_i < \widehat{p}_{-i}) = \Pr (p_i < p(c_1)) \Pr (p_i < p(c_2)) .... \Pr (p_i < p(c_n)). \tag{3}$$

Let us denote with $p^{-1}(p_j)$ the marginal cost that firm j must have in order to select price p_j. Clearly, our interest is limited to the values of p_j for which

$$0 \leq p^{-1}(p_j) \leq 1. \tag{4}$$

Since the marginal cost c_j is uniformly distributed on $[0, 1]$,

$$\Pr (p_i < p(c_j)) = \Pr \left(p^{-1}(p_i) < c_j\right) = 1 - p^{-1}(p_i) \tag{5}$$

and (3) can be rewritten:

$$\Pr (p_i < \widehat{p}_{-i}) = \left(1 - p^{-1}(p_i)\right)^{n-1}. \tag{6}$$

The maximisation problem (2) becomes

$$\begin{cases} \max\limits_{p_i} (1 - p_i)(p_i - c_i)(1 - p^{-1}(p_i))^{n-1} \\ \text{s.t. } 0 \leq p^{-1}(p_i) \leq 1. \end{cases} \tag{7}$$

The reader can easily verify that the unique corner solution is given by $p_i = p^*$ where p^* is the price chosen by any rival that uses the strategy $p(.)$ when its cost of production is zero. However, this would be an equilibrium only in the particular event in which all the firms have a unit cost equal to zero. Since I assumed that the costs are uniformly and independently distributed, the probability of this event is zero. Therefore, when looking for a Bayesian-Nash equilibrium of our game one should ignore this corner solution.

The first order necessary condition for an interior optimum is that

$$(-2p_i+c_i+1)\left(1\text{-}p^{-1}(p_i)\right)^{n-1}+\left(-p_i^2+p_ic_i+p_i\text{-}c_i\right)\frac{\partial}{\partial p_i}\left(1\text{-}p^{-1}(p_i)\right)^{n-1}=0. \tag{8}$$

The first order condition defines an implicit function of player i's best response to the strategy $p(.)$ played by all the rival firms, given that player i's marginal cost is c_i. If the strategy $p(.)$ is to be a symmetric Bayesian-Nash equilibrium, we require that the solution to the first order condition be $p(c_i)$: that is, for each firm i's possible marginal costs, firm i does not wish to deviate from the strategy $p(.)$ given that i's rivals play this strategy. To impose this requirement, we substitute $p_i = p(c_i)$ into the first order condition, obtaining (also taking into account that $p^{-1}[p(c_i)] = c_i$):

$$p'(c_i)(1-c_i)(1+c_i-2p(c_i))-(n-1)(p(c_i)-c_i)(1-p(c_i))=0. \tag{9}$$

Equation (9) can be rewritten in the following way:

$$p'(c_i)=\frac{(n-1)(p(c_i)-c_i)(1-p(c_i))}{(1-c_i)(1+c_i-2p(c_i))}. \tag{10}$$

The following lemma is very useful to determine the unique Bayesian-Nash equilibrium of our game.

Lemma 1. *The unique strictly monotone function which solves the differential equation in (10) is*

$$p_i(c_i):=\frac{1}{1+n}+\frac{n}{1+n}c_i. \tag{11}$$

Proof. See Appendix 1.

From Lemma 1, the following Proposition follows quite easily:

Proposition 1. *The strategy defined by equation (11) is the unique symmetric Bayesian-Nash equilibrium of the game defined by assumptions A1-A6.*

Proof. See Appendix 2.

Since all firms adopt the same strictly increasing function of the cost of production, the firm that can produce at the minimum cost will charge the smallest price. Moreover, since $Pr(c_i = c_j) = 0$, there will be only one firm charging the minimum price. Therefore, this firm will satisfy the whole market demand. Now, supposing, without loss of generality that firm 1 has the minimum cost of production, the Bertrand equilibrium price will be

$$p^B = \frac{1}{1+n} + \frac{n}{1+n} c_1 \tag{12}$$

and the equilibrium quantity will be

$$Q^B = q_1^B = 1 - p^B = n\frac{1-c_1}{1+n}. \tag{13}$$

Note that, even though only one firm produces in equilibrium, all firms have a positive *ex ante* expected profit before the game is actually played. Indeed,

$$\overline{\Pi}_i^B = (1-p_i)(p_i - c_i)\Pr(p_i < \widehat{p}_{-i}) = n\frac{(1-c_i)^{n+1}}{(1+n)^2} \tag{14}$$

which is equal to zero only in the extreme case where $c_i = 1$ and strictly positive elsewhere.

Note also that the price charged by each firm is always greater than the unit cost of production and, hence, the firm that ends up operating always obtains positive profits. More precisely, the profit of the firm that ends up operating is

$$\Pi_1^B = \left(1 - p^B\right)\left(p^B - c_1\right) = n\frac{(1-c_1)^2}{(1+n)^2}. \tag{15}$$

which is equal to zero only in the extreme case where $c_1 = 1$, which occurs with probability zero, and strictly positive elsewhere. Clearly, all the other firms, i.e. the less efficient ones, will obtain, *ex post*, a profit equal to zero.

Finally, note that the price, the *ex ante* expected profit of every firm and the *ex post* profit of the winner are decreasing functions of the number of firms in the industry and if this number tends to infinity, then each firm tends to choose a price equal to its unit cost and the *ex ante* and *ex post* profits all converge to zero.

2.2 The Cournot game.

The Cournot Bayesian-Nash equilibrium when rivals' costs are unknown is more straightforward to derive than the corresponding Bertrand Bayesian-Nash equilibrium analysed in the previous subsection. In particular, Cramton and Palfrey (1990) calculate this equilibrium by making assumptions A1-A6. In what follows I give the results that they obtain. The interested reader will find the detailed proof in the original paper. In this game, each firm maximises its expected profit given the output decisions of the other firms.

Firm i's expected profit is

$$\overline{\Pi}_i^C = (p - c_i)\, q_i = (\widetilde{c} - q_i - c_i)\, q_i, \tag{16}$$

where $\widetilde{c} = 1 - (n-1)\overline{q}$ and $\overline{q}$ is the expected value of q_i.

Cramton and Palfrey show that, given assumptions A1-A6, there exists a unique Bayesian-Nash equilibrium in which the quantity of output produced by firm i with cost c_i is

$$q_i^C = \begin{cases} 0 & \text{if } c_i \geq \widetilde{c} \\ \frac{\widetilde{c} - c_i}{2} & \text{if } c_i < \widetilde{c} \end{cases} \tag{17}$$

where $\widetilde{c} = 1 - (n-1)\overline{q} = \frac{2}{\sqrt{n}+1}$.

Therefore, the resulting Cournot equilibrium price is

$$p^C = 1 - \sum_{i=1}^{n} q_i^C \tag{18}$$

and the Cournot equilibrium aggregate output is

$$Q^C = \sum_{i=1}^{n} q_i^C . \tag{19}$$

Firm i's *ex ante* expected profit is

$$\overline{\Pi}_i^C = \begin{cases} 0 & \text{if } c_i \geq \widetilde{c} \\ \frac{(\widetilde{c}-c_i)^2}{4} & \text{if } c_i < \widetilde{c} \end{cases} . \tag{20}$$

Firm i's *ex post* profit is

$$\Pi_i^C = \begin{cases} 0 & \text{if } c_i \geq \widetilde{c} \\ \left(p^C - c_i\right) \frac{(\widetilde{c}-c_i)}{2} & \text{if } c_i < \widetilde{c} \end{cases} . \tag{21}$$

3 Comparison of the two models.

We now have all the necessary material to compare the two models. For the sake of simplicity in this section I assume that in the industry there are only two firms, that is $n = 2$.[2]

In what follows I show that Bertrand competition can lead to a higher (lower) price (quantity) than Cournot competition. Moreover, in the Bertrand game, firm i's *ex ante* expected profit is always higher and also the *ex post* profit of the more efficient firm is very likely to be higher than in the Cournot game.

[2]For simplicity, I make the assumption that $n = 2$. If $n > 2$, the qualitative results shown below would not change.

3.1 Equilibrium Prices and Quantities.

The market price resulting from the Bertrand game can be higher than the one resulting from the Cournot game. Correspondingly, the aggregate Bertrand quantity, consumer surplus and total surplus, will be lower than the corresponding Cournot values. This means that Cournot competition can lead to more efficient outcomes than Bertrand competition.

Let the costs of the two firms be c_1 and c_2 and suppose, without loss of generality, that $c_1 < c_2$. From (12) we know that if the two firms engage in the Bertrand game, the resulting market price is

$$p^B = \frac{1+2c_1}{3}. \tag{22}$$

To the contrary, (17) and (18) lead to the following market price in the Cournot game:

$$p^C = 1 - Q^C = \begin{cases} 3 - 2\sqrt{2} + \frac{c_1+c_2}{2} & \text{if } c_1 < c_2 < 2\left(\sqrt{2}-1\right) \\ 2 - \sqrt{2} + \frac{c_1}{2} & \text{if } c_1 < 2\left(\sqrt{2}-1\right) \le c_2 \\ 1 & \text{if } 2\left(\sqrt{2}-1\right) \le c_1 < c_2 \end{cases}, \tag{23}$$

where $Q^C = q_1^C + q_2^C$ is the aggregate Cournot output.

The reader can easily verify that $p^B \geq p^C$ whenever the costs of the two firms satisfy the following condition:

$$c_1 < c_2 \leq \frac{c_1 + 4(3\sqrt{2}-4)}{3} \tag{24}$$

which implies that $c_1 < 2\left(3\sqrt{2}-4\right) \simeq 0.485$.

Recalling that the costs of production are uniformly and independently distributed on [0, 1], we can conclude that the condition in (24) will be satisfied with a probability equal to 15.7% (see Figure 1 where $c' = 2\left(3\sqrt{2}-4\right)$).

Hence, if the two firms are relatively efficient, the Bertrand game will lead to a higher price than the Cournot game. The intuition of this result is as follows. In the Cournot game, firms use (17) and, therefore, if both have low costs, they will both produce a relatively large quantity of output leading to a relatively low market price. To the contrary, in the Bertrand game, due to the assumptions that goods are perfect substitutes and firms have no capacity constraints, only the firm with the lowest cost of production will produce in equilibrium and will satisfy the whole market demand given that the chosen price is the one in (22). Now, even though the price is strictly increasing in the cost of production, the price-cost margin is strictly decreasing with respect to the same variable. That is, like in the monopoly case, the market price is relatively higher when the cost of production is relatively low.

Note also that since price and aggregate quantity are inversely related (i.e. $p = 1 - Q$), whenever the Bertrand price is higher than the Cournot price, the Bertrand aggregate output will be smaller. This means that with a probability of 15.7% the static Cournot game with incomplete information will lead to an outcome which is more efficient in welfare terms than the corresponding Bertrand game.

3.2 *Ex ante* expected profits.

We will now see that the *ex ante* expected profit for both firms is higher in the Bertrand game than in the Cournot game.

From (14), firm i's *ex ante* Bertrand profit when $n = 2$ is

$$\overline{\Pi}_i^B = \frac{2(1-c_i)^3}{9}. \tag{25}$$

To the contrary, from (20) firm i's *ex ante* Cournot profit when $n = 2$ is

$$\overline{\Pi}_i^C = \begin{cases} 0 & \text{if } c_i \geq 2\left(\sqrt{2}-1\right) \\ \frac{\left(2\left(\sqrt{2}-1\right)-c_i\right)^2}{4} & \text{if } c_i < 2\left(\sqrt{2}-1\right) \end{cases}. \tag{26}$$

The reader can easily check that $\overline{\Pi}_i^B \geq \overline{\Pi}_i^C$ when $c_i \in [0,1]$. That is, the *ex ante* expected profit from engaging in price competition against the rival is higher than the one from engaging in quantity competition and this is true whatever the values of the two costs of production are.

As I mentioned in the Introduction, this result contradicts Proposition 9 in Vives (1984) where the opposite statement is derived in a duopoly model where firms have private information about an uncertain linear demand.

3.3 *Ex post* profits.

In the Bertrand game, the *ex post* profit of the less efficient firm (i.e. firm 2) is always zero. To the contrary, in the Cournot game, this will be zero if $c_2 \geq 2\left(\sqrt{2}-1\right)$ and positive otherwise. Things are different for the more efficient firm. Indeed, we will now see that the *ex post* profit of firm 1 is very likely to be higher in the Bertrand game than in the Cournot one.

From (15), the *ex post* Bertrand profit of the winner when $n = 2$ is

$$\Pi_1^B = \frac{2\left(1-c_1\right)^2}{9}. \tag{27}$$

To the contrary, from (21) and (23), the *ex post* Cournot profit of the firm with the minimum cost when $n = 2$ is

$$\Pi_1^C = \begin{cases} \left(3 - 2\sqrt{2} + \frac{c_2 - c_1}{2}\right) \frac{2\left(\sqrt{2}-1\right) - c_1}{2} & \text{if } c_1 < c_2 < 2\left(\sqrt{2}-1\right) \\ \left(2 - \sqrt{2} - \frac{c_1}{2}\right) \frac{2\left(\sqrt{2}-1\right) - c_1}{2} & \text{if } c_1 < 2\left(\sqrt{2}-1\right) \leq c_2 \\ 0 & \text{if } 2\left(\sqrt{2}-1\right) \leq c_1 < c_2 \end{cases} . \tag{28}$$

The reader can verify that $\Pi_1^B \geq \Pi_1^C$ whenever the costs of the two firms satisfy the following condition:

$$\begin{cases} c_2 \in [c_1, \widehat{c}] & \text{if } c_1 \leq c' \\ c_2 \in [c_1, 1] & \text{if } c_1 > c' \end{cases}, \tag{29}$$

where

$$c' = 2\left(3\sqrt{2} - 4\right),$$

$$\widehat{c} = \frac{-c_1^2 + \left(20 - 18\sqrt{2}\right)c_1 + 260 - 180\sqrt{2}}{9\left(2\sqrt{2} - 2 - c_1\right)}.$$

Again, since the costs of production are uniformly and independently distributed on $[0,1]$, we can conclude that the condition in (29) will be satisfied with a probability equal to 77.942% (see Figure 2 for a graphical representation). That is, as I have already mentioned, the Bertrand game will lead with a very high probability to a higher profit for the more efficient firm than the Cournot game. This means that profits in Bertrand competition can be higher than in Cournot competition even when goods are substitutes and not only when they are complements as stated by Singh and Vives (1984) in the complete information framework.

A 1

Proof of Lemma 1. Consider the differential equation in (10), which is relabelled as (A1):

$$p'(c_i) = \frac{(1-n)(c_i - p(c_i))(1 - p(c_i))}{(c_i - 1)(2p(c_i) - c_i - 1)} \tag{A1}$$

The function $p'(c_i)$ is everywhere continuous except for all points (c_i, p_i) where either $c_i = 1$ or $p_i = (c_i + 1)/2$. Representing these lines on a graph (see Figure A1) and recalling that $c_i \in [0,1]$, we can infer that there exist two different regions to the left of $c_i = 1$ where $p'(c_i)$ is continuous: the one below the line $p(c_i) = (c_i + 1)/2$ and the one above that same line. In these two regions, the conditions required by the Cauchy-Peano theorem for the existence and uniqueness of a solution of a differential equation like the one in (A1) hold (see, for example R. Grimshaw, 1990, Chap.1).

This means that in each of the above mentioned regions there exists a unique solution.

Let us see now whether these solutions can be linear.

By substituting in (A1) the generic function $p(c_i)$ with the function

$$p_i = a + bc_i$$

we obtain:

$$c_i^2(nb^2\text{-}nb+b^2)+c_i(\text{-}2b^2\text{-}1+a+b+2nab\text{-}an\text{-}bn+n)+na^2\text{-}na\text{-}2ab+b\text{-}a^2+a=0 \tag{A2}$$

The values of a and b that solve equation A2) are

$$(a, b) = (1, 0)$$

and

$$(a, b) = \left(\frac{1}{1+n}, \frac{n}{1+n}\right).$$

Therefore, in the region above the line $p(c_i) = (c_i + 1)/2$, the unique solution is given by the line $p_i = 1$. However, this solution is irrelevant for the solution of our problem since I assumed that the strategy $p(.)$ adopted by i's rivals has to be a strictly monotone function of the unit cost of production. To the contrary, in the region below the line $p(c_i) = (c_i + 1)/2$, the unique solution is given by the line $p_i = \frac{1}{1+n} + \frac{n}{1+n}c_i$.

Q.E.D.

A 2

Proof of Proposition 1. From (6) and (11) we obtain

$$\Pr(p_i < \widehat{p}_{-i}) = \left[\frac{(n+1)(1-p_i)}{n}\right]^{n-1} \text{ with } \frac{1}{1+n} \leq p_i \leq 1. \qquad \text{(A3)}$$

Therefore, if all firms except firm i adopt strategy (11), then problem (2) can be rewritten in the following way:

$$\begin{cases} \max\limits_{p_i} (1-p_i)^n (p_i - c_i) \left(\frac{n+1}{n}\right)^{n-1} \\ \text{s.t. } \frac{1}{1+n} \leq p_i \leq 1. \end{cases}, \qquad \text{(A4)}$$

whose unique solution is (11).

Therefore, strategy (11) is a Bayesian-Nash equilibrium of our game. Moreover, since Lemma 1 states that there do not exist other strategies that satisfy the first order conditions, strategy (11) is also the unique Bayesian-Nash equilibrium of the game.

Q.E.D.

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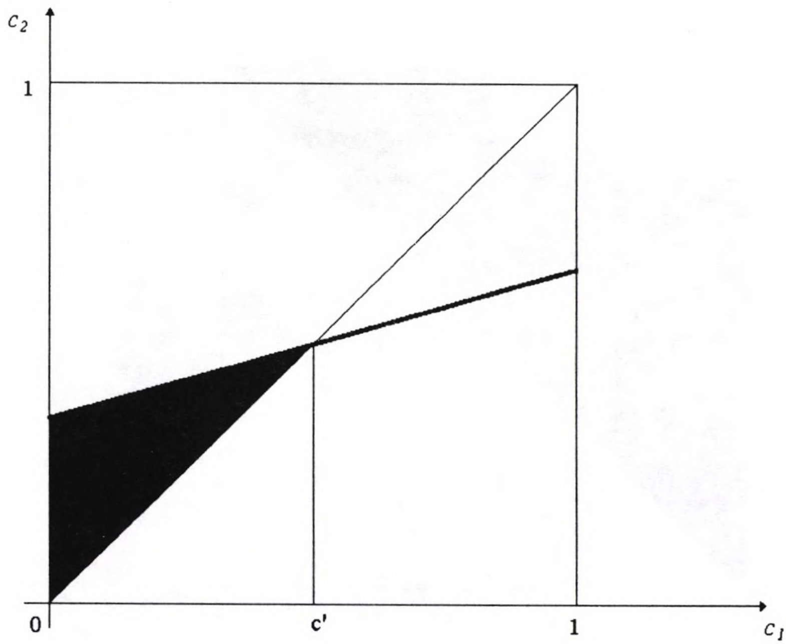

Figure 1. The costs in the shaded region are such that $p^B \geq p^C$.

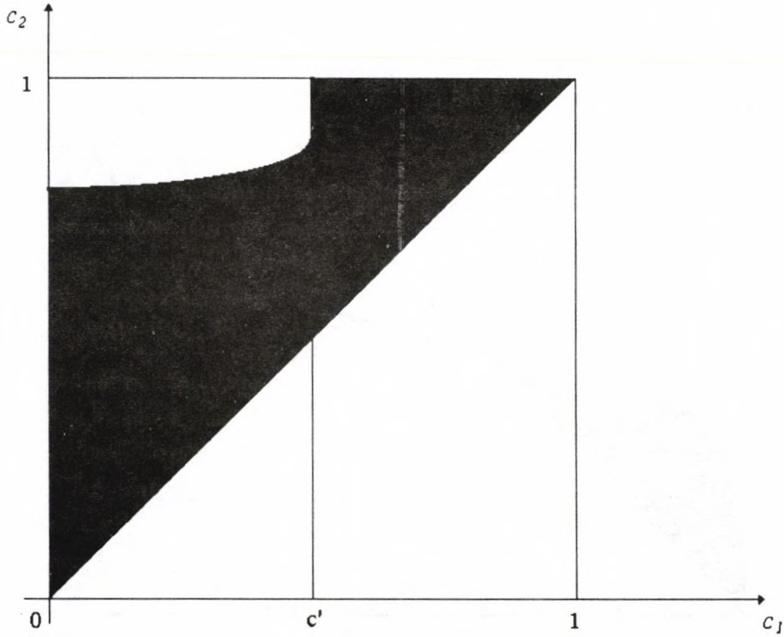

Figure 2. The costs in the shaded region are such that $\Pi_1^B \geq \Pi_1^C$.

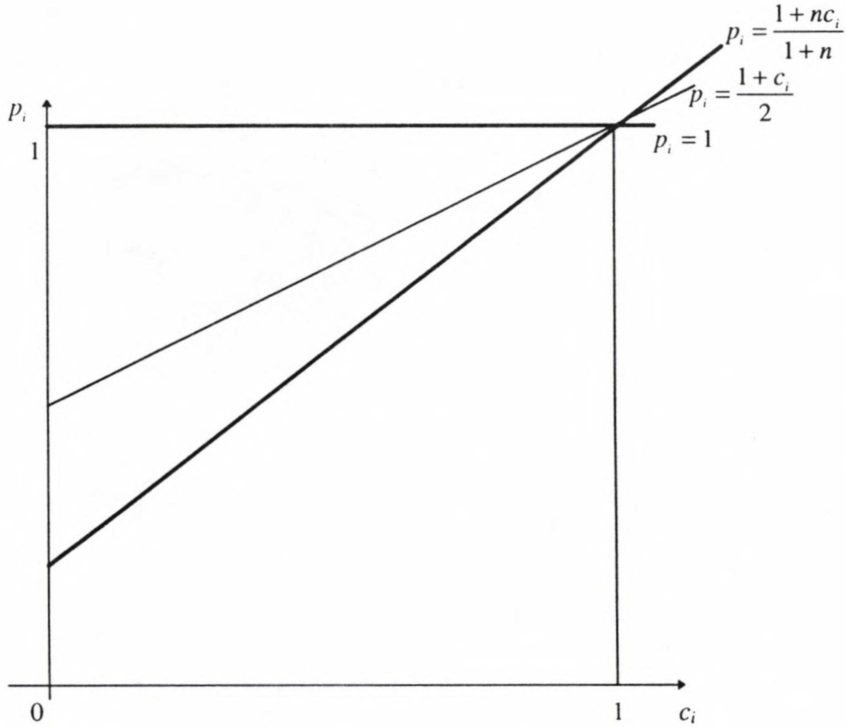

Figure A1. The bold lines represent the two solutions to equation A1.

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